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$$\therefore P = \frac{\pi a^2 bc^2 lmn \sin B}{2\sqrt{[(ablm+acln+bcmn)^3]}} = \frac{\pi abc \triangle lmn}{\sqrt{[(ablm+acln+bcmn)^3]}}.$$

But $al+bm+cn = 2\triangle$.

$$\therefore P = \frac{\pi \triangle acln[2\triangle - al - cn]}{\sqrt{[(2\triangle al + 2\triangle cn - acln - a^2l^2 - c^2n^2)^3]}}.$$

From the equation to the ellipse, we get

$$BD = u = \frac{acn}{bm+cn} = \frac{acn}{2\triangle - al}, \quad BF = v = \frac{acl}{al+bm} = \frac{acl}{2\triangle - cn}.$$

$$\therefore al = \frac{2\triangle v(a-u)}{ac-uv}, \quad cn = \frac{2\triangle u(c-v)}{ac-uv}.$$

$$\therefore P = \frac{\pi \triangle uv \sqrt{[(a-u)(c-v)]}}{\sqrt{[(av+cu-uv)^3]}}. \quad \text{Let } a-u=t, \ c-v=z.$$

$$\therefore P = \frac{\pi \triangle [a-t][c-z]\sqrt{[tz]}}{\sqrt{[(ac-tz)^3]}}.$$

$$\therefore Q = \int_0^a \int_0^c P \, dt \, dz \Big/ \int_0^a \int_0^c dt \, dz = \frac{\pi \triangle}{ac} \int_0^a \int_0^c \frac{(a-t)(c-z)\sqrt{(tz)} \, dt \, dz}{\sqrt{[(ac-tz)^3]}}.$$

Let $tz = ac\sin^2\theta$, $\theta' = \sin^{-1}\sqrt{(t/a)}$.

$$\therefore Q = \frac{2\pi\triangle}{a} \int_0^a \int_0^{\theta'} \frac{(a-t)(t-a\sin^2\theta)\sin^2\theta \, dt \, d\theta}{t^2\cos^2\theta}$$

$$= \frac{\pi\triangle}{a} \int_0^a \left(\frac{a-t}{t^2}\right)\{3a\sin^{-1}\sqrt{(t/a)} - 2t\sin^{-1}\sqrt{(t/a)} - 3\sqrt{[t(a-t)]}\}dt.$$

Let $t = a\sin^2\varphi$.

$$\therefore Q = 2\pi\triangle \int_0^{\frac{1}{4}\pi} (3\varphi - 2\varphi\sin^2\varphi - 3\sin\varphi\cos\varphi)\cot^3\varphi \, d\varphi = \frac{1}{2}\pi^2\triangle(7-10\log^2)$$

127. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

What is the probable error of the volume of a rectangular parallelopiped whose edges measured by the repeated application of a unit of measure are found to be a, b, c, supposing that the probable error of a line so measured whose length is found to be l is $r\sqrt{l}$?

Solution by the PROPOSER.

The probable error for a, $=r\sqrt{a}$; for b, $r\sqrt{b}$; for c, $r\sqrt{c}$.

$\therefore$ Error of volume in length$=bcr\sqrt{a}$.

Error of volume in width$=acr\sqrt{b}$.

Error of volume in thickness$=abr\sqrt{c}$.

The probable error of volume$=$square root of the sum of the squares of these three errors.

$\therefore$ Probable error$=\sqrt{[(ab^2c^2+a^2bc^2+a^2b^2c)r^2]}=r\sqrt{[abc(ab+ac+bc)]}$.

MISCELLANEOUS.

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124. Proposed by **J. W. YOUNG**, Graduate Student, Cornell University, Ithaca, N. Y.

Prove that the general value of θ, which satisfies the equation

$$(\cos\theta+i\sin\theta)(\cos2\theta+i\sin2\theta)\ldots\text{to } n \text{ factors}=1 \text{ is } \frac{4m\pi}{n(n+1)};$$

where m is any integer $(i=\sqrt{-1})$.

Solution by **G. W. GREENWOOD, A. M.**, McKendree College, Lebanon. Ill.; **LON C. WALKER, A. M.**, Leland Stanford Jr. University, Cal., and **J. SCHEFFER, A. M.**, Hagerstown, Md.

$$1=(\cos\theta+i\sin\theta)(\cos2\theta+i\sin2\theta)\ldots(\cos n\theta+i\sin n\theta)=(\cos\theta+i\sin\theta)^{1+2+\cdots+n}$$

$$=(\cos\theta+i\sin\theta)^{\frac{1}{2}[n(n+1)]}=\cos\frac{n(n+1)\theta}{2}+i\sin\frac{n(n+1)\theta}{2}.$$

$$\therefore \frac{n(n+1)\theta}{2}=2m\pi, \text{ where } m \text{ is any integer; } i.\ e.\ \theta=\frac{4m\pi}{n(n+1)}.$$

Also solved by *G. B. M. ZERR.*

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125. Proposed by **F. P. MATZ, Sc. D., Ph. D.**, Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Assume $m=nt+\varepsilon-\omega$, thus giving $v=m+e\sin v$ as the relation connecting the mean and eccentric anomalies, then express $x=a\cos v$, $y=b\sin v$, and $r=a(1-e\cos v)$ by a Fourier series in terms of m.

Solution by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $y_1=z+x\varphi((y)$, we get by Lagrange's Theorem,

$$f(y_1)=f(z)+x\varphi(z)f'(z)+\frac{x^2}{1.2}\frac{d}{dz}\{[\varphi(z)]^2f'(z)\}+\frac{x^3}{1.2.3}\left(\frac{d}{dz}\right)^2\{[\varphi(z)]^3f'(z)\}+$$

$$\text{etc., etc.}$$

From $v=m+e\sin v$, $y_1=v$, $z=m$, $x=e$, $\varphi(y)=\sin v$.

Now $f(v)=v$ and $f'(v)=1$.